Finite Automata: Homework 1

1. We define

   $f(0) = 0,\ f(1) = 1,\ f(n+2) = f(n)$

   $g(0) = 0,\ g(n+1) = 1 - g(n)$

   Compute $f(2), f(3), g(1), g(2), g(3)$. Prove that $f(n) = g(n)$ for all n

2. Design DFA for each of the following set

   (a) the set of words in $\{4, 8, 1\}$ containing the subword 481

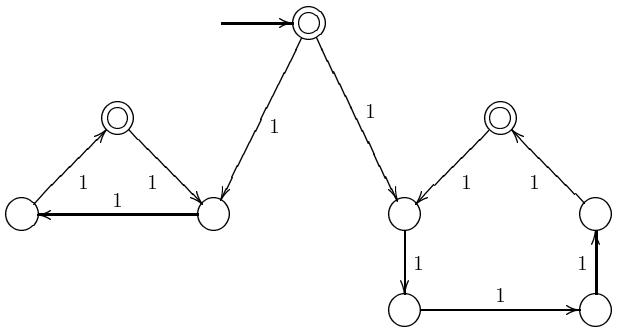
   (b) the set of words in $\{0, 1\}$ with an even number of occurences of 0 and a number of occurences of 1 divisible by 3

   (c) the set of words in $\{0\}$ whose length is divisible by either 2 or 3

3. Show that $(ab)^n a = a(ba)^n$ holds for all natural number n.

4. Let $f : \Sigma^* \to \Theta^*$ be such that $f(\epsilon) = \epsilon$ and $f(xy) = f(x)f(y)$ for all $x, y \in \Sigma^*$. Show that if $L \subseteq \Theta^*$ is regular then so is $f^{-1}(L) = \{x \in \Sigma^* \mid f(x) \in L\}$.

5. Let Σ be $\{1\}$. The following NFA



   accepts all words of length multiple of 3 *or* 5. Using the subset construction, build a DFA accepting the same language. Try then to understand intuitively how this DFA is working.